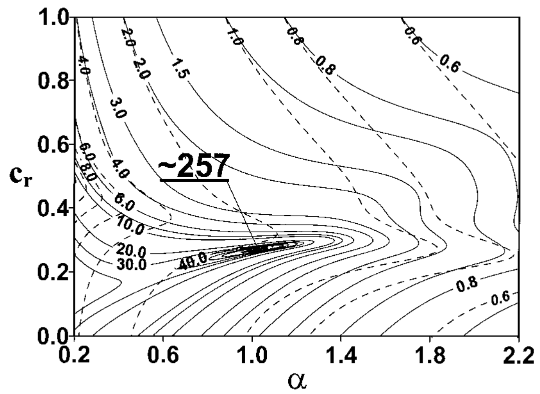

Fig. 5 Variations of the magnitude of the flow response $\|(u, v)\|$ as a function of the suction wave number α and wave speed c_r for $Re = 5 \times 10^3$; continuous and dashed lines correspond to the symmetric and asymmetric suctions, respectively.

eigensolution (TS wave) and a unique asymmetric particular solution of the inhomogeneous problem. A very large response of the flow is possible in this case but requires a perfect tuning between the neutral TS wave and the surface suction wave.

Results presented in Fig. 3 demonstrate that linear theory is inadequate in the neighborhood of (linear) neutral stability points and a nonlinear theory must be used regardless of the magnitude of the suction. The nonlinear effect may expand the range of the TS instability and may initiate an instability of the type studied in Ref. 1. Figure 5 shows flow response to surface suction for the subcritical value of Reynolds number $Re = 5 \times 10^3$ when all TS waves are stable. The near resonance between the TS and the surface suction waves may lower the critical Reynolds number due to the subharmonic character of the instability, provided that suction with sufficiently large amplitude is used.

IV. Conclusions

The preceding results demonstrate that small suction nonuniformities may affect the flow instability only if they contain the critical or near-critical suction waves, that is, waves with the same or almost the same wave number α and phase speed c_r as the neutral TS waves. Suction waves slightly detuned with the neutral TS waves may affect instability provided that the amplitude of suction nonuniformities is large enough. Our conclusions could be readily tested in a flow with a few suction slots at the wall that could emulate suction waves with the desired wave number and phase speed.

Acknowledgments

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Vibration of Thermally Stressed Pretwisted Cantilever Composite Plates

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I. Introduction

PRETWISTED cantilever composite plates are used extensively as blades in engines and turbomachinery and are very important structural members. Therefore their vibration characteristics are important for efficient sizing and design. Composite pretwisted plates are integral parts of engines of high-performance aircraft and naval structures. These are envisioned to operate at very high or low temperatures. Supersonic and hypersonic flight can be noted in this regard. This applies equally well to other structures operating in hostile environments. It thus appears imperative to study the vibration characteristics of pretwisted composite plates that are thermally stressed.

Studies concerning the vibration of blades as well as isotropic and composite twisted cantilever plates have appeared in the literature.[1–5] In addition, researchers are studying the free vibration of composite laminated plates subjected to temperature changes.[6–8] The purpose of the present work is to study the variation of the fundamental natural frequency for pretwisted cantilever composite plates for various laminations at different temperatures and angles of twist.

II. Computational Experiments

Figure 1 illustrates the composite pretwisted cantilever plate; also shown are the geometric and material data. Two symmetric and two asymmetric eight-layer laminations are considered, namely, 1) $(0/90/0/90)_s$, 2) $(45/-45/0/90)_s$, 3) $(30/-30)_4$, and

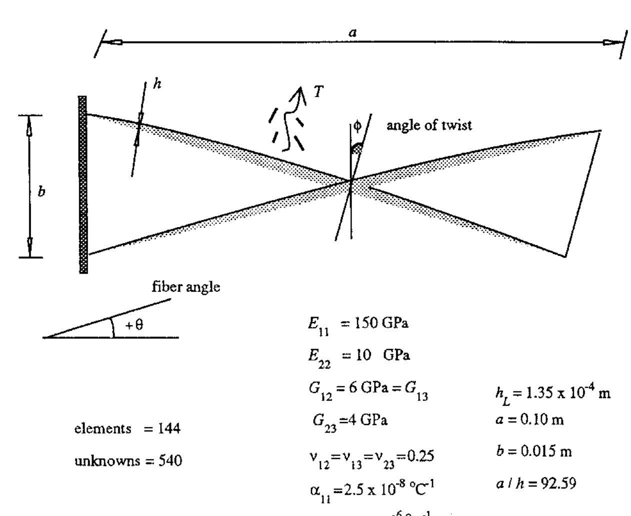

Fig. 1 Geometrical and material data for a pretwisted cantilever composite plate.

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4) $(30/-60/0/0)_2$. The pretwisted plate is discretized with a set of triangular shell finite elements that are based on the natural-mode finite element method.[9,10] The validation of the element on free vibration of cantilever plates was reported in a previous work[11] by comparison with other results as well as experimental data. The finite element mesh included 144 triangular elements for a total of 540 degrees of freedom. Using this mesh, a convergent solution is obtained. We consider a temperature range between 0 and 150°C. This temperature is applied in steps in the context of a full geometrically nonlinear analysis. At convergence for the imposed temperature, the fundamental vibration frequency is extracted. At this point, the composite plate is fully thermally stressed.

Table 1 shows the comparison of the first frequency obtained by the present element at $T = 0$ and of a higher-order theory[12] for a simply supported quasi-isotropic laminate for various aspect ratios. Figure 2 presents the frequency–temperature curves for the four aforementioned laminates and three angles of twist. All curves are nonlinear with increasing temperature. In all cases the increase in temperature and angle of twist leads in stiffening of the structure, which raises the fundamental frequency except for the quasi-isotropic $(45/-45/0/90)_s$ laminate whose fundamental frequency at 50°C and angle of twist 90 deg is lower than the corresponding frequencies at 0- and 45-deg angles of twist, respectively. Although the $(45/-45/0/90)_s$ laminate exhibits the lowest vibration frequency at $T = 0$°C, with the increase in temperature its fundamental frequency surpasses those of the $(30/-30)_4$ and $(30/-60/0/0)_2$ plates. This shows that the temperature increase impacts the vibration characteristics of certain laminates to a greater extent than others. Overall the laminates with an angle of twist equal to 90 deg exhibit the highest natural frequencies at all temperatures. For all angles of twist considered, the $(0/90/0/90)_s$ laminate had the highest fundamental natural frequency at all temperatures, and the $(30/-60/0/0)_2$ plate the lowest. Interestingly enough, the fundamental frequencies of the $(0/90/0/90)_s$ and $(45/-45/0/90)_s$ laminates at 0- and 45-deg angles of twist remain very close in the interval 50–100°C, whereas they get raised with subsequent increase in temperature. A similar remark can be made for the 90-deg pretwisted $(0/90/0/90)_s$ laminate. Note that, at 50°C, the frequencies of the $(45/-45/0/90)_s$, $(30/-30)_4$, and $(30/-60/0/0)_2$ plates are essentially coincident. Figure 3 shows the first vibration mode for the $(45/-45/0/90)_s$ 90-deg pretwisted plate at $T = 0$°C, and Fig. 4 depicts the first mode

Table 1 Comparison of the first frequency at $T = 0$ of present element and of higher-order theory for a simply supported eight-layer $(0/45/-45/90)_s$ laminate for various aspect ratios $d = l/h$

$d = l/h$	Present	Higher-order theory[12]
10	16.0978	16.193
20	18.9759	19.100
50	20.1143	20.323
100	20.2963	20.559

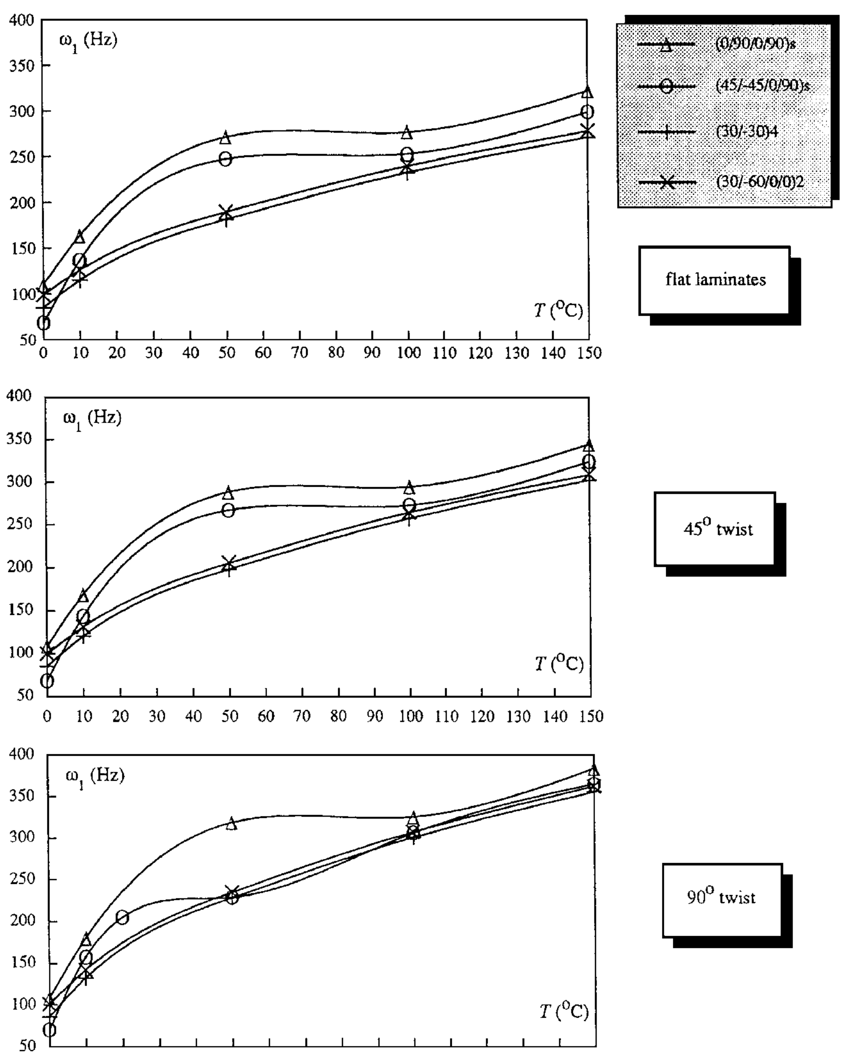

Fig. 2 Frequency–temperature curves for four flat and pretwisted composite laminates.

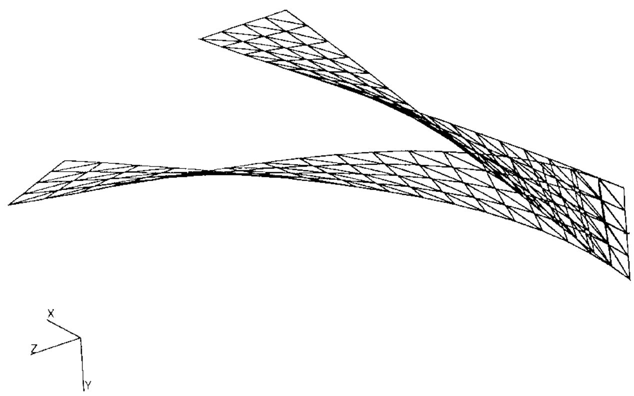

Fig. 3 Fundamental vibration mode for the $(45/-45/0/90)_s$ 90-deg pretwisted cantilever laminated plate at $T = 0°C$.

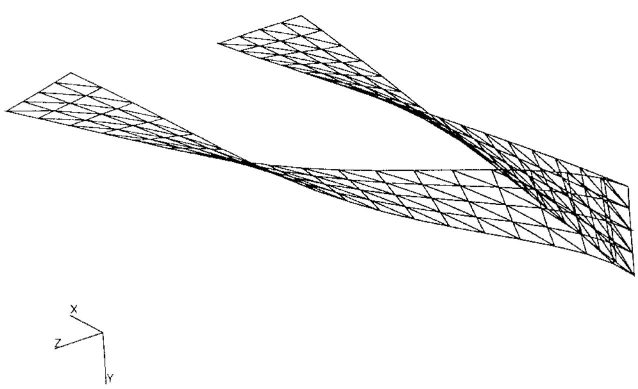

Fig. 4 Fundamental vibration mode for the $(45/-45/0/90)_s$ 90-deg pretwisted cantilever laminated plate at $T = 150°C$.

for the same laminate and twist at $T = 150°C$. The effect of thermal stressing evidently changes the vibration mode.

From the results presented, one immediately deduces the interesting nonlinear characteristics of the temperature–frequency curves of pretwisted composite laminated plates. It will also be useful to produce the same curves for the same plates but with temperature-dependent material properties. This must be taken into account when higher temperatures are considered. Of course, it also depends on the material used. This will be the subject of a future study.

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